

GROUP THEORY MEETS GRAPH THEORY: A SYSTEMATIC AND BIBLIOMETRIC REVIEW OF ALGEBRAIC GRAPH STRUCTURES AND EMERGING APPLICATIONS

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ABSTRACT

The interaction between group theory and graph theory has developed from classical representations of algebraic structures into a broad interdisciplinary research domain encompassing algebraic graph theory, spectral analysis, computational group theory, network science, quantum information and symmetry-aware artificial intelligence. This article presents a systematic and bibliometric review of research connecting group-theoretic structures with graphs, with emphasis on Cayley, Schreier, power, commuting and non-commuting graphs, graph automorphisms, spectral methods and representation-theoretic approaches. It further examines the transition of these mathematical structures toward computational and interdisciplinary applications, including complex networks, biomedical systems, cryptography, quantum computing, geometric deep learning, molecular modelling and materials science. The review combines systematic-review principles, mathematical taxonomy and a reproducible bibliometric protocol. The evidence window is restricted to literature published or publicly available on or before 31 May 2025. The synthesis shows that algebraic graph theory operates as a bidirectional interface: algebraic structures generate graphs, graph properties reveal algebraic structure, and matrix- and representation-theoretic methods mediate between the two. The paper proposes an evolutionary framework progressing from group structure and group actions through graph symmetry, algebraic graph construction, computational analysis, network representation and symmetry-aware learning. It identifies research opportunities in approximate symmetry, dynamic algebraic graphs, scalable computation, mathematically interpretable graph learning, quantum graph computation and scientific AI. A six-layer Algebraic Graph Intelligence framework is proposed to connect algebraic foundations with intelligent graph-based applications.

Keywords: group theory; algebraic graph theory; Cayley graphs; Schreier graphs; power graphs; commuting graphs; graph automorphism; spectral graph theory; representation theory; bibliometrics; graph neural networks; equivariant learning; quantum computing; network science

1. INTRODUCTION

Group theory and graph theory developed from different mathematical traditions, yet their interaction has produced a productive interface within discrete mathematics. Group theory provides a language for operations, transformations and symmetry, while graph theory provides a language for relationships, connectivity and combinatorial structure. Their intersection becomes especially powerful when an algebraic structure can generate a graph or when graph symmetry can be characterized through an algebraic group.

For a group G and generating subset S , a Cayley graph $\Gamma = \text{Cay}(G, S)$ uses elements of G as vertices and group multiplication to determine adjacency. The construction makes generators graphically visible and gives the graph a regular group action. The relationship extends beyond Cayley graphs: Schreier graphs encode group actions on coset spaces; power graphs encode power relations; commuting and non-commuting graphs encode algebraic relations; and automorphism groups describe graph symmetries.

Algebraic graph theory therefore should not be understood as a one-directional application of algebra to graphs. It is a bidirectional framework in which algebraic structures generate graphs, graph invariants characterize algebraic structures, and matrix- and representation-theoretic techniques mediate between the two. Foundational treatments such as Biggs (1993) and Godsil and Royle (2001) establish the mathematical breadth of this interface.

The field has also undergone a computational transformation. Algorithms for permutation groups, graph isomorphism and automorphism computation, together with systems such as GAP, allow researchers to investigate large families of algebraic graphs experimentally as well as theoretically. At the same time, applications have expanded toward network science, cryptography, quantum information, molecular modelling and symmetry-aware machine learning.

This review consequently frames the evolution as a progression from group structure and group actions to graph symmetry, algebraic graph construction, computational analysis, network representation and equivariant learning. The purpose is not to equate these domains, but to identify the mathematical connections and the research opportunities that arise at their interfaces.

2. REVIEW OBJECTIVES AND RESEARCH QUESTIONS

2.1 Research objectives

- RO1.** Examine the theoretical evolution linking group theory, group actions and algebraic graph theory.
- RO2.** Classify principal algebraic graph structures, including Cayley, Schreier, power, commuting, non-commuting and automorphism-related graphs.
- RO3.** Identify the intellectual, thematic, geographical and collaborative structure of the research field through bibliometric analysis.
- RO4.** Investigate the transition from classical structural problems toward computational and interdisciplinary applications.
- RO5.** Identify emerging research themes, unresolved problems and future research opportunities at the intersection of group theory, graph theory and intelligent computational systems.

2.2 Research questions

- RQ1.** How has research connecting group theory and graph theory evolved over time?
- RQ2.** Which group-theoretic concepts have been most influential in algebraic graph theory?
- RQ3.** What are the principal algebraic graph structures investigated in the literature?
- RQ4.** What intellectual and thematic clusters characterize the field?
- RQ5.** Which authors, institutions, countries and publication sources have contributed substantially to the field?
- RQ6.** How have computational methods changed the investigation of group-associated graph structures?
- RQ7.** Which emerging applications connect algebraic graph theory with contemporary computational and interdisciplinary research?
- RQ8.** What research gaps and future directions emerge from the systematic and bibliometric evidence?

3. METHODOLOGY

3.1 Review design

The study adopts an integrated systematic-review, bibliometric and mathematical-synthesis design. The methodology combines six analytical activities: systematic literature identification; mathematical classification; bibliometric performance analysis; science mapping; thematic synthesis; and research-gap identification. PRISMA 2020 principles are used to structure identification, screening, eligibility and reporting (Page et al., 2021).

Systematic review + Bibliometrics + Science mapping + Mathematical taxonomy + Research-gap analysis

3.2 Databases and search strategy

Scopus is proposed as the primary bibliographic database because of its multidisciplinary coverage and exportable citation metadata. Web of Science Core Collection is recommended as a secondary validation source. Crossref, publisher databases, MathSciNet and zbMATH Open may be used for bibliographic verification, while Google Scholar should be treated as supplementary rather than equivalent to the primary bibliometric corpus.

The search strategy combines group-theoretic and graph-theoretic terms. A reproducible master search string is given below.

TITLE-ABS-KEY(("group theory" OR "group action*" OR "permutation group*" OR "automorphism group*" OR "representation theory") AND ("algebraic graph*" OR "Cayley graph*" OR "Schreier graph*" OR "power graph*" OR "commuting graph*" OR "non-commuting graph*" OR "vertex-transitive graph*" OR "graph automorphism*"))

3.3 Eligibility criteria

- Include studies that substantively address group-theoretic graph structures, group actions on graphs, graph automorphisms, algebraic graph families, spectral or representation-theoretic methods, computational approaches, or documented interdisciplinary applications.
- Where terminology differs across disciplines, use backward and forward citation checking to reduce false-negative retrieval.

3.4 Screening and data extraction

The screening sequence is: identification, duplicate removal, title and abstract screening, full-text eligibility, and final inclusion. The final counts must be taken directly from the actual database exports. No publication, citation, author or cluster count is estimated in this manuscript.

For each included record, the extraction sheet should contain publication year, authors, title, source, DOI, citation count, affiliations, countries, keywords, abstract, mathematical structure, group concept, methodological approach and application domain.

3.5 Bibliometric analysis

Bibliometrix (Aria & Cuccurullo, 2017) is suitable for comprehensive science mapping, while VOSviewer (van Eck & Waltman, 2010) is suitable for network visualization. Recommended analyses include annual publication growth, citation structure, source productivity, author and country analysis, co-authorship, co-citation, bibliographic coupling, keyword co-occurrence, thematic evolution and three-field analysis.

$$CPP = \text{Total citations} / \text{Total publications} \quad (1)$$

$$BC_{ij} = |R_i \cap R_j| \quad (2)$$

Co-citation analysis is used to identify intellectual foundations, whereas bibliographic coupling is used to identify research fronts. Keyword co-occurrence is used to identify conceptual clusters. The analytical settings — counting method, occurrence threshold, normalization, clustering method and resolution — should be reported to ensure reproducibility.

3.6 Methodological safeguard

Bibliometric prominence should not be equated with mathematical importance. Citation counts are affected by publication age, database coverage and disciplinary citation practices. Likewise, a bibliometric cluster is an empirical property of a selected corpus rather than an objective mathematical category. These distinctions are essential when interpreting results.

4. MATHEMATICAL FOUNDATIONS

4.1 Group theory

A group $(G, *)$ is an algebraic structure satisfying closure, associativity, identity and inverse properties. Its relevance to graph theory is rooted in the ability of group elements to define structured transformations. If G acts on X , each $g \in G$ induces $x \mapsto g \cdot x$, and the orbit of x is $Gx = \{g \cdot x : g \in G\}$. When X is the vertex set of a graph, the action becomes a mechanism for describing graph symmetry.

4.2 Group actions on graphs

Let $G \leq \text{Aut}(\Gamma)$. Every element of G then preserves adjacency. For a vertex v , the stabilizer is $G_v = \{g \in G : g(v) = v\}$. For finite groups, the orbit–stabilizer relationship holds as given in Equation (3). Vertex-transitivity, edge-transitivity and arc-transitivity can therefore be formulated through group actions.

$$G \leq \text{Aut}(\Gamma), \quad |Gv| = |G| / |G_v| \quad (3)$$

4.3 Automorphisms and classification

The automorphism group $\text{Aut}(\Gamma)$ consists of all permutations of $V(\Gamma)$ that preserve adjacency. It provides a natural algebraic descriptor of graph symmetry. Classification can therefore combine combinatorial parameters, automorphism groups, permutation actions, spectra and underlying algebraic structures.

$$\Gamma \rightarrow \text{Aut}(\Gamma) \rightarrow \text{permutation-group structure} \rightarrow \text{classification}$$

Cayley graphs form a proper subclass of vertex-transitive graphs: every Cayley graph is vertex-transitive, but not every vertex-transitive graph is a Cayley graph. Stronger forms of symmetry include arc-transitivity and distance-transitivity.

4.4 Spectral graph theory

For adjacency matrix A , degree matrix D , Laplacian L and signless Laplacian Q , one has $L = D - A$ and $Q = D + A$. Eigenvalues provide structural information and, for algebraically generated graphs, may be interpreted through group representations. Spectral information should nevertheless not be treated as a complete graph-isomorphism invariant, because non-isomorphic graphs can be cospectral.

$$\text{Spec}(\Gamma) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}, \quad L = D - A, \quad Q = D + A \quad (4)$$

4.5 Representation theory

A representation is a homomorphism $\rho : G \rightarrow GL(V)$. In algebraic graph theory, representations connect group operations with matrices and operators. This gives the conceptual chain shown below. Modern equivariant learning later extends the same principle by constraining neural representations to transform consistently under specified groups.

$$\text{Group} \rightarrow \text{Representation} \rightarrow \text{Matrix} \rightarrow \text{Graph spectrum}$$

5. EVOLUTION OF ALGEBRAIC GRAPH STRUCTURES

5.1 Cayley graphs

For a group G and inverse-closed generating set S , the Cayley graph $\text{Cay}(G, S)$ has vertex set G and adjacency $g \sim h$ whenever $g^{-1}h \in S$. Its degree is $|S|$, and connectedness is equivalent to $\langle S \rangle = G$. The right-regular action of G provides a regular subgroup of $\text{Aut}(\text{Cay}(G, S))$. Cayley-graph isomorphism, normal Cayley graphs and CI-groups form important structural themes (Li, 2002).

$$\text{Cay}(G, S): V = G, \quad g \sim h \Leftrightarrow g^{-1}h \in S \quad (5)$$

5.2 Schreier graphs

Schreier graphs generalize Cayley graphs by representing group actions on sets such as coset spaces G/H . They provide a natural connection among subgroups, permutation representations and graph structure. Cayley graphs arise as the special case $H = \{e\}$.

5.3 Power graphs

In a power graph, vertices are group elements and two vertices are adjacent when one is an integer power of the other. The construction exposes element orders, cyclic subgroups and subgroup relationships, and it supports questions concerning connectivity, automorphisms, spectra and graph reconstruction.

5.4 Commuting and non-commuting graphs

For a non-abelian group G , a commuting graph uses the relation $xy = yx$ as adjacency, while a non-commuting graph uses $xy \neq yx$. Centralizers $C_G(x)$ therefore become structural objects in the graph. These graph families translate non-abelian algebraic behaviour into combinatorial properties.

$$C_G(x) = \{g \in G : gx = xg\} \quad (6)$$

5.5 Other algebraically defined graphs

The broader literature includes graphs constructed from rings and other algebraic structures, including zero-divisor and total graphs. Recent reviews of graphs associated with rings illustrate the breadth of this extension. Such constructions reinforce a general principle: an algebraic relation can be converted into graph adjacency, after which graph invariants can be used to study the originating structure.

6. STRUCTURAL CLASSIFICATION AND COMPUTATIONAL TRANSFORMATION

6.1 Symmetry hierarchy

$$\text{Regular} \rightarrow \text{Vertex-transitive} \rightarrow \text{Edge-transitive} \rightarrow \text{Arc-transitive} \rightarrow \text{Distance-transitive}$$

These properties are related but not interchangeable. Classification through $\text{Aut}(\Gamma)$ can examine order, transitivity, vertex stabilizers, primitive or imprimitive actions, regular subgroups and Cayley representations.

6.2 Strongly regular graphs

A strongly regular graph is characterized by parameters (v, k, λ, μ) . Its adjacency matrix satisfies the identity in Equation (7). This illustrates the recurring transition from combinatorial regularity to algebraic constraints and classification.

$$A^2 = (k - \mu)I + (\lambda - \mu)A + \mu J \quad (7)$$

6.3 Computational group theory

Computational group theory makes it possible to determine automorphism groups, stabilizers and orbits, to generate algebraic graphs and to test isomorphism algorithmically. GAP is particularly useful for finite and permutation-group calculations. A reproducible computational study should report the group specification, generating set, graph construction, software environment, algorithms, parameters, calculated invariants and source code where possible.

$$G \rightarrow S \rightarrow \text{Cay}(G, S) \rightarrow \text{Aut}(\Gamma) \rightarrow \text{invariants} \rightarrow \text{classification}$$

6.4 Computational reproducibility

- (i) Specify the algebraic structure and its representation.
- (ii) Specify the graph construction and generating set.
- (iii) Report software, version and algorithms.
- (iv) Report graph parameters and calculated invariants.
- (v) Provide source code and computational data where possible.

7. EMERGING APPLICATIONS

7.1 Complex networks

Real-world networks are heterogeneous and noisy, unlike ideal algebraic graphs. Exact automorphisms may therefore be rare. Research on network symmetry motivates approximate and stochastic notions of symmetry. Orbit partitions can still provide a mathematically precise notion of structural equivalence when an appropriate group action exists.

$$\text{Exact symmetry} \rightarrow \text{approximate equivalence} \rightarrow \text{stochastic symmetry}$$

7.2 Cryptography

Group-based cryptography has investigated computational problems associated with non-abelian groups, including conjugacy, decomposition, membership and word problems. Braid and polycyclic groups have been studied as platforms for cryptographic constructions (Fine et al., 2011; Gryak & Kahrobaei, 2016). However, algebraic complexity is not synonymous with cryptographic hardness. Security depends on assumptions, parameter choices, known attacks and implementation.

The review therefore treats group-based cryptography as an application and research intersection rather than claiming automatic post-quantum security for non-abelian constructions.

7.3 Biomedical networks

Biological systems can be represented through protein–protein interaction, gene-regulatory, metabolic, signalling and disease networks. Algebraic graph methods can contribute symmetry, automorphism, matrix and structural-equivalence perspectives. Rietman et al. (2011) explicitly examined group theory and algebraic graph methods in molecular systems biology. Mathematical symmetry should

nevertheless not be interpreted automatically as biological functional equivalence; empirical validation remains necessary.

7.4 Quantum computing

Group representations occur throughout quantum information through unitary representations, Lie groups, quantum Fourier transforms, stabilizer structures and symmetry-constrained computation. Quantum walks provide another bridge between graphs and quantum dynamics. Rau (2021) reviews the role of symmetry and geometry in qubit systems.

$$Group \rightarrow Representation \rightarrow Quantum\ transformation \rightarrow Quantum\ computation$$

7.5 Artificial intelligence and equivariant graph learning

Modern geometric deep learning provides a direct conceptual bridge between groups, graphs, geometry and machine learning (Bronstein et al., 2021). For a group action, an invariant function satisfies $f(gx) = f(x)$, while an equivariant function satisfies $F(gx) = gF(x)$. For a graph adjacency matrix transformed by a permutation P , $A' = PAP^T$, and a graph-level invariant satisfies $f(A) = f(PAP^T)$. These principles connect classical symmetry theory with graph neural architectures.

$$f(gx) = f(x) \quad [\text{invariance}] \tag{8}$$

$$F(gx) = gF(x) \quad [\text{equivariance}] \tag{9}$$

$$A' = PAP^T, \quad f(A) = f(PAP^T) \tag{10}$$

7.6 Chemistry, molecular modelling and materials science

Molecules and materials are governed by geometric symmetries. Under a rigid transformation $x_i \mapsto Rx_i + t$, scalar energy should satisfy $E(RX + t) = E(X)$, while vector-valued forces should transform as $F(RX + t) = RF(X)$. Recent equivariant graph models demonstrate how such symmetry constraints can be incorporated into molecular-property prediction and molecular-dynamics modelling (Nehil-Puleo et al., 2024).

$$E(RX + t) = E(X), \quad F(RX + t) = R F(X) \tag{11}$$

8. INTEGRATED TAXONOMY OF ALGEBRAIC GRAPH STRUCTURES

Table 1 consolidates the principal constructions reviewed in Sections 4 to 7, relating each algebraic input to its graph construction, its dominant structural signal and its potential application domain.

Table 1. Integrated taxonomy of algebraic graph structures.

Algebraic input	Graph construction	Principal structural signal	Potential application
Group + generators	Cayley graph	Regularity, transitivity, expansion, spectrum	Networks, coding, computation
Group action + subgroup or set	Schreier graph	Orbits, stabilizers, permutation action	Group actions, geometry
Group + power relation	Power graph	Element orders, cyclic subgroups	Algebraic reconstruction
Group + commutativity	Commuting graph	Centralizers, clique structure	Finite-group characterization
Group + non-commutativity	Non-commuting graph	Non-abelian interaction structure	Group classification
Group + automorphisms	Symmetry graph or action	Orbits, stabilizers, transitivity	Structural classification

Algebraic input	Graph construction	Principal structural signal	Potential application
Representation + matrix	Spectral or algebraic graph	Eigenvalues, invariant subspaces	Spectral analysis
Algebraic structure + geometry	Equivariant graph model	Invariance and equivariance	Scientific AI

9. BIBLIOMETRIC ANALYSIS FRAMEWORK

The bibliometric component is designed to quantify the intellectual and thematic structure of the field after retrieval of the actual Scopus or Web of Science corpus. The present manuscript deliberately does not invent publication counts, citation totals, rankings or network clusters.

9.1 Performance analysis

- Annual publication output and growth rate
- Total citations and citations per year
- Source productivity
- Author productivity
- Country and institutional participation
- Co-authorship patterns

9.2 Science mapping

- Co-citation mapping to identify intellectual foundations
- Bibliographic coupling to identify contemporary research fronts
- Keyword co-occurrence to identify conceptual clusters
- Thematic evolution to examine temporal shifts
- Three-field analysis linking authors, keywords and sources

9.3 Anticipated analytical phases

The literature synthesis suggests four broad periods that should be tested empirically rather than treated as pre-established bibliometric findings: foundational group and symmetry work; structural expansion involving Cayley, vertex-transitive, spectral and computational themes; specialization in power, commuting and related algebraic graphs; and interdisciplinary expansion involving artificial intelligence, quantum, molecular, biomedical and network-science themes.

10. RESEARCH GAPS AND FUTURE RESEARCH AGENDA

10.1 Fragmentation of graph families

Cayley, Schreier, power, commuting and other graph families are often investigated separately. A comparative framework could study a family of graphs associated with the same algebraic object.

10.2 Structure-to-application disconnect

Classical work emphasizes classification, connectivity, spectra, automorphisms and isomorphism, while applications emphasize prediction, optimization, learning and simulation. A bridge is needed from mathematical invariant to computational representation and application-level feature.

10.3 Representation theory and graph learning

Classical representations $\rho : G \rightarrow GL(V)$ provide a natural theoretical foundation for modern equivariant architectures. The relationship deserves systematic mathematical and empirical investigation.

10.4 Exact versus approximate symmetry

Empirical networks and noisy data rarely exhibit perfect group symmetry. Approximate and stochastic group actions therefore represent an important frontier.

10.5 Dynamic algebraic graphs

Most classical constructions are static. Dynamic formulations could represent evolving biological, communication, infrastructure and molecular systems.

10.6 Mathematical explainability

Automorphism groups, orbit structures, centralizers, generators and spectra may provide interpretable descriptors for graph-learning models.

10.7 Quantum–algebraic graph learning

Cayley, strongly regular and vertex-transitive graphs provide structured benchmark families for investigating quantum graph representations and quantum learning.

10.8 Biomedical validation

Algebraic structural equivalence requires empirical biological validation before functional interpretation.

10.9 Unified computational benchmarks

Standard benchmarks should compare graph-isomorphism runtime, automorphism computation, classification accuracy, spectral descriptors, equivariant-learning robustness, molecular prediction error and quantum-resource requirements.

11. Proposed Algebraic Graph Intelligence Framework

The synthesis proposes an Algebraic Graph Intelligence framework as a conceptual bridge between classical mathematics and emerging computational systems. The framework does not claim that graph neural networks constitute algebraic graph theory; rather, it identifies symmetry and representation as common mathematical interfaces. Its six layers are summarized in Table 2.

Table 2. Six-layer Algebraic Graph Intelligence framework.

Layer	Core elements
Layer 1 — Algebraic foundation	Groups, subgroups, homomorphisms and representations
Layer 2 — Symmetry	Group actions, stabilizers, orbits and automorphisms
Layer 3 — Graph construction	Cayley, Schreier, power, commuting and related graphs
Layer 4 — Structural computation	Isomorphism, automorphism, spectra, classification and symbolic computation
Layer 5 — Representation and learning	Graph embeddings, graph neural networks, invariant and equivariant learning
Layer 6 — Intelligent application	Networks, biology, quantum systems, chemistry, materials and prediction

Algebra → Symmetry → Graph → Computation → Learning → Intelligence

The framework supports a shift in perspective from symmetry as an object of mathematical investigation, to symmetry as a computational method and, increasingly, as an inductive bias embedded within learning architectures.

12. INTEGRATED DISCUSSION

The synthesis indicates that the relationship between groups and graphs is bidirectional. A group can generate a graph, while graph structure can reveal properties of the originating algebraic object. This is visible in Cayley constructions, automorphism analysis and representation-theoretic spectral methods.

The computational transformation is equally important. Algebraic graph research increasingly operates through a theory–algorithm–computation cycle rather than a purely theorem-oriented workflow. This creates opportunities for experimental mathematics, automated classification and reproducible computational studies.

The application layer should be interpreted with discipline. Cayley graphs are highly regular mathematical architectures and should not be treated as generic models of empirical networks. Similarly, non-abelian group structure does not itself establish cryptographic security, and mathematical symmetry does not itself establish biological functional equivalence.

The strongest contemporary bridge appears in symmetry-aware machine learning. Group invariance and equivariance translate mathematical transformation laws into constraints on representations and neural architectures. Molecular and materials modelling provides particularly clear examples, because physical laws impose transformation requirements on energies, forces and geometric configurations.

Taken together, the literature suggests an evolutionary movement from symmetry as object, to symmetry as method, to symmetry as computational inductive bias. This interpretation offers a coherent way to connect classical algebraic graph theory with modern network science, quantum information and scientific artificial intelligence without collapsing distinct research traditions into one field.

13. LIMITATIONS

- Database coverage does not represent the entire mathematical literature; important monographs, specialist journals and proceedings may be underrepresented.
- Bibliometric indicators are affected by publication age, database coverage and disciplinary citation practices.
- Terminological fragmentation across mathematics, computer science, physics and biomedical science may create false negatives.
- The proposed application links should not be interpreted as evidence of validated domain performance unless supported by empirical studies.
- Bibliometric numerical results are intentionally not fabricated. Final publication counts, citation metrics and network maps require the actual database export and analysis.

CONCLUSION

The interaction between group theory and graph theory has evolved from classical algebraic descriptions of symmetry toward an interdisciplinary ecosystem involving structural classification, computation, networks, quantum information, scientific modelling and equivariant artificial intelligence. At its mathematical core are groups, group actions, automorphisms, representations and

spectra. These concepts generate and explain graph structures including Cayley, Schreier, power, commuting and non-commuting graphs.

The central contribution of this review is an evolutionary interpretation in which algebraic graph theory functions as an interface rather than an isolated collection of graph families. The progression can be summarized as follows.

*Group theory → Graph symmetry → Algebraic graph theory → Computational graph analysis →
Equivariant learning → Graph intelligence*

Future work should concentrate on approximate symmetry, dynamic algebraic graphs, scalable computation, representation-aware learning, mathematical explainability, quantum graph computation and validated scientific applications. The proposed Algebraic Graph Intelligence framework provides a conceptual basis for organizing these directions while retaining the distinction between established mathematical theory and emerging interdisciplinary applications.

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APPENDIX A. BIBLIOMETRIC REPORTING CHECKLIST

The following items should be reported in full when the empirical bibliometric results are added to this manuscript.

- Database name and access date
- Exact search string
- Search fields
- Publication date range and the 31 May 2025 cut-off
- Number of records retrieved
- Duplicate-removal method and count
- Title and abstract screening count
- Full-text eligibility count
- Final included corpus
- Software and version
- Counting method
- Thresholds
- Normalization method
- Clustering method
- Network-map interpretation procedure
- Data availability statement

APPENDIX B. DATA-INTEGRITY STATEMENT

This manuscript intentionally separates conceptual synthesis from empirical bibliometric results. Any final submission reporting publication counts, citation totals, author rankings, institutional rankings, country rankings, co-citation clusters, keyword maps or thematic-evolution statistics should insert those values only after the underlying Scopus or Web of Science corpus has been retrieved, screened and analysed. This safeguard prevents unsupported or simulated bibliometric findings.